

Fig. 3. Deflection ratio as function of  $b_2/b_1$  for several values of  $\beta$ .

$$M = \int_{-H_t}^{H_c} \sigma_y y b \, dy$$

After substitution for  $\sigma$  and  $b$  and integration this becomes

$$\frac{3}{b_1 H^3} \frac{M H_c}{\sigma_{H_c}} = h_c^3 + \frac{1}{4} \left( \frac{b_2}{b_1} - 1 \right) h_c^4 + \beta \left[ h_t^3 + \left( \frac{b_2}{b_1} - 1 \right) h_c h_t^3 + \frac{3}{4} \left( \frac{b_2}{b_1} - 1 \right) h_t^4 \right] \quad (2)$$

Similarly for this section shown in Fig. 1(b),

$$\frac{3}{b_1 H^3} \frac{M H'_c}{\sigma'_{H_c}} = \beta \left[ h'_t{}^3 + \frac{1}{4} \left( \frac{b_2}{b_1} - 1 \right) h'_t{}^4 \right] + h'_c{}^3 + \left( \frac{b_2}{b_1} - 1 \right) h'_c{}^3 h'_t + \frac{3}{4} \left( \frac{b_2}{b_1} - 1 \right) h'_c{}^4 \quad (3)$$

With the preceding equations, the bending stresses may be determined for a given cross section and bending moment if the parameter  $\beta$  is known. As  $\beta$  has to be determined, however, the equations must be used in somewhat different form.

In pure bending, the radius of curvature  $R$  of the neutral axis at a given time is related to the surface strain by  $1/R = \epsilon_{H_c}/H_c = \beta \sigma_{H_c} F/H_c$  and similarly  $1/R' = \beta \sigma'_{H'_c} F/H'_c$ . Hence the ratio of the two curvatures at a given time  $1/R \div 1/R'$  is merely the right-hand side of Eq. (3) divided by the right-hand side of Eq. (2). The curvature is given by  $1/R = d^2y/dx^2/[1 + (dy/dx)^2]^{3/2}$ , where  $y$  is bending deflection and  $x$  is distance along the beam. In most practical cases the slope  $dy/dx$  is small enough for this to be written as  $1/R = d^2y/dx^2$  and bending deflections will then be directly proportional to  $1/R$ . Thus, if the beams with cross sections shown in Fig. 1 are subjected to the same bending moments the ratio of deflections  $\delta/\delta'$  at corresponding locations along the beam at a given time are given by

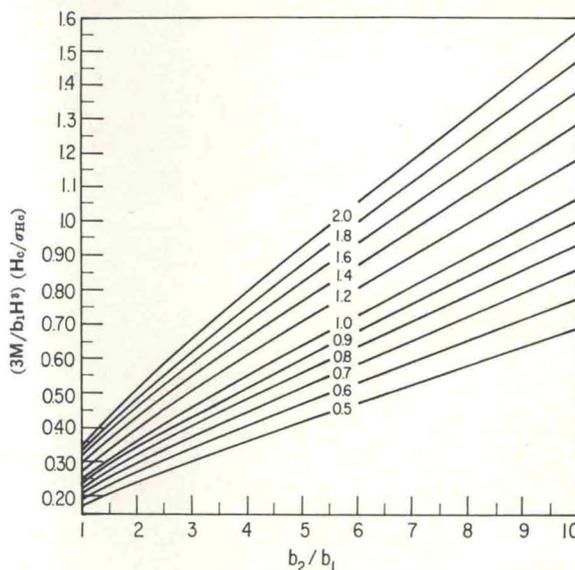


Fig. 4. Quantity used to obtain creep constants as a function of  $b_2/b_1$  for several values of  $\beta$ .

$$\delta/\delta' = \frac{1/R}{1/R'} = \frac{\text{Right-hand side Eq. (3)}}{\text{Right-hand side Eq. (2)}}$$

This relation is shown in Fig. 3 for a range of values of  $\beta$  and  $b_2/b_1$ . By observing the ratio at several times, for which creep strains are large relative to elastic strains, the ratio  $\beta$  may be found. If  $\beta$  varies greatly with time the preceding analysis is not applicable, whereas if  $\beta = 1$ , tension and compression creep are equivalent and the analysis of bending tests presents no difficulty. If  $\beta \neq 1$ , the next step is to determine the individual tension or compression creep data. This may be done by using Eq. (2) to find  $\sigma_{H_c}$  for a given bending moment. Measurements of curvature as a function of time (obtained from bending deflection data) then make it possible to determine  $F$  from  $F = (1/R)(H_c/\sigma_{H_c})(1/\beta)$ . For this purpose Fig. 4 shows  $(3M/b_1H^3)(H_c/\sigma_{H_c})$  for various values of  $\beta$  and  $b_2/b_1$ .

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